Roger Cotes

Roger Cotes FRS (10 July 1682 – 5 June 1716) was an English mathematician, known for working closely with Isaac Newton by proofreading the second edition of his famous book, the *Principia*, before publication. He also devised the quadrature formulas known as Newton–Cotes formulas, which originated from Newton's research,[4] and made a geometric argument that can be interpreted as a logarithmic version of Euler's formula.[5] He was the first Plumian Professor at Cambridge University from 1707 until his death.

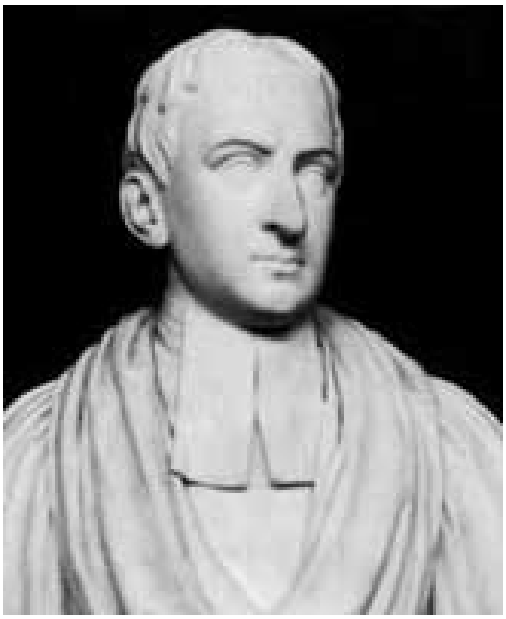
Early life

Cotes was born in Burbage, Leicestershire. His parents were Robert, the rector of Burbage, and his wife, Grace, *née* Farmer. Roger had an elder brother, Anthony (born 1681), and a younger sister, Susanna (born 1683), both of whom died young. At first Roger attended Leicester School, where his mathematical talent was recognised. His aunt Hannah had married Rev. John Smith, and Smith took on the role of tutor to encourage Roger's talent. The Smiths' son, Robert Smith, became a close associate of Roger Cotes throughout his life. Cotes later studied at St Paul's School in London and entered Trinity College, Cambridge, in 1699.[6] He graduated BA in 1702 and MA in 1706.[2]

Astronomy

Roger Cotes's contributions to modern computational methods lie heavily in the fields of astronomy and mathematics. Cotes began his educational career with a focus on astronomy. He became a fellow of Trinity College in 1707, and at age 26 he became the first Plumian Professor of Astronomy and Experimental Philosophy. On his appointment to professor, he opened a subscription list in an effort to provide an observatory

Roger Cotes
FRS



This bust was commissioned by Robert Smith and sculpted posthumously by Peter Scheemakers in 1758.

Born	10 July 1682 Burbage, Leicestershire, England
Died	5 June 1716 (aged 33) Cambridge, Cambridgeshire, England
Alma mater	Trinity College, Cambridge
Known for	Lituus spiral Logarithmic spiral Least squares Euler's formula proof Concept of the radian Cotes spiral Newton–Cotes formulas
Scientific career	
Fields	Mathematician
Institutions	Trinity College, Cambridge

for Trinity. Unfortunately, the observatory was still unfinished when Cotes died, and was demolished in 1797.[2]

Academic advisors	Isaac Newton Richard Bentley[1]
Notable students	Robert Smith[2] James Jurin[3] Stephen Gray

In correspondence with Isaac Newton, Cotes designed a heliostat telescope with a mirror revolving by clockwork.[7][8] He recomputed the solar and planetary tables of Giovanni Domenico Cassini and John Flamsteed, and he intended to create tables of the moon's motion, based on Newtonian principles. Finally, in 1707 he formed a school of physical sciences at Trinity in partnership with William Whiston.[2]

The *Principia*

From 1709 to 1713, Cotes became heavily involved with the second edition of Newton's *Principia*, a book that explained Newton's theory of universal gravitation. The first edition of *Principia* had only a few copies printed and was in need of revision to include Newton's works and principles of lunar and planetary theory.[2] Newton at first had a casual approach to the revision, since he had all but given up scientific work. However, through the vigorous passion displayed by Cotes, Newton's scientific hunger was once again reignited. The two spent nearly three and half years collaborating on the work, in which they fully deduce, from Newton's laws of motion, the theory of the moon, the equinoxes, and the orbits of comets. Only 750 copies of the second edition were printed[2] although pirated copies from Amsterdam were also distributed to meet the demand for the work. As a reward to Cotes, he was given a share of the profits and 12 copies of his own. Cotes's original contribution to the work was a preface which supported the scientific superiority of Newton's principles over the then popular vortex theory of gravity advocated by René Descartes. Cotes concluded that the Newton's law of gravitation was confirmed by observation of celestial phenomena that were inconsistent with the vortex theory.[2]

Mathematics

Cotes's major original work was in mathematics, especially in the fields of integral calculus, logarithms, and numerical analysis. He published only one scientific paper in his lifetime, titled *Logometria*, in which he successfully constructs the logarithmic spiral.[9][10] After his death, many of Cotes's mathematical papers were edited by his cousin Robert Smith and published in a book, *Harmonia mensurarum*.[2][11] Cotes's additional works were later published in Thomas Simpson's *The Doctrine and Application of Fluxions*.[9] Although Cotes's style was somewhat obscure, his systematic approach to integration and mathematical theory was highly regarded by his peers. Cotes discovered an important theorem on the n-th roots of unity,[12] foresaw the method of least squares,[13] and discovered a method for integrating rational fractions with binomial denominators.[9][14] He was also praised for his efforts in numerical methods, especially in interpolation methods and his table construction techniques.[9] He was regarded as one of the few British mathematicians capable of following the powerful work of Sir Isaac Newton.

Death and assessment

Cotes died from a violent fever in Cambridge in 1716 at the early age of 33. Isaac Newton remarked, "If he had lived we would have known something."[2]

See also

- Cotes's spiral
- Extended Euclidean algorithm
- Newton–Cotes formulas
- Lituus (mathematics)

References

1. Gowing 2002, p. 5.
2. Meli (2004)
3. Rusnock (2004) "Jurin, James (bap. 1684, d. 1750) (http://www.oxforddnb.com/view/article/15173)", *Oxford Dictionary of National Biography*, Oxford University Press, retrieved 6 September 2007 (subscription, Wikipedia Library (https://wikipedialibrary.wmflabs.org/partners/88/) access or UK public library membership (https://www.oxforddnb.com/help/subscribe#public) required)
4. Iliffe, Rob; Smith, George E., eds. (2016). *The Cambridge Companion to Newton* (2nd ed.). Cambridge University Press. p. 411. doi:10.1017/cco9781139058568 (https://doi.org/10.1017%2Fcco9781139058568). ISBN 978-1-139-05856-8.

5. Cotes wrote: *"Nam si quadrantis circuli quilibet arcus, radio* CE *descriptus, sinun habeat* CX *sinumque complementi ad quadrantem* XE*; sumendo radium* CE *pro Modulo, arcus erit rationis inter* $EX + XC\sqrt{-1}$ *&* CE *mensura ducta in* $\sqrt{-1}$*."* (Thus if any arc of a quadrant of a circle, described by the radius *CE*, has sinus *CX* and sinus of the complement to the quadrant *XE*; taking the radius *CE* as modulus, the arc will be the measure of the ratio between $EX + XC\sqrt{-1}$ & *CE* multiplied by $\sqrt{-1}$.) That is, consider a circle having center *E* (at the origin of the (x, y) plane) and radius *CE*. Consider an angle *θ* with its vertex at *E* having the positive x-axis as one side and a radius *CE* as the other side. The perpendicular from the point *C* on the circle to the x-axis is the "sinus" *CX*; the line between the circle's center *E* and the point *X* at the foot of the perpendicular is *XE*, which is the "sinus of the complement to the quadrant" or "cosinus". The ratio between $EX + XC\sqrt{-1}$ and *CE* is thus $\cos\theta + \sqrt{-1}\sin\theta$. In Cotes' terminology, the "measure" of a quantity is its natural logarithm, and the "modulus" is a conversion factor that transforms a measure of angle into circular arc length (here, the modulus is the radius (*CE*) of the circle). According to Cotes, the product of the modulus and the measure (logarithm) of the ratio, when multiplied by $\sqrt{-1}$, equals the length of the circular arc subtended by *θ*, which for any angle measured in radians is *CE* • *θ*. Thus, $\sqrt{-1}CE\ln\left(\cos\theta + \sqrt{-1}\sin\theta\right) = (CE)\theta$. This equation has the wrong sign: the factor of $\sqrt{-1}$ should be on the right side of the equation, not the left. If this change is made, then, after dividing both sides by *CE* and exponentiating both sides, the result is: $\cos\theta + \sqrt{-1}\sin\theta = e^{\sqrt{-1}\theta}$, which is Euler's formula.
   See:
   - Roger Cotes (1714) "Logometria," *Philosophical Transactions of the Royal Society of London*, **29** (338) : 5-45; see especially page 32. Available on-line at: HathiTrust (http://babel.hathitrust.org/cgi/pt?id=ucm.5324351035;view=2up;seq=38)
   - Roger Cotes with Robert Smith, ed., *Harmonia mensurarum* … (Cambridge, England: 1722), chapter: "Logometria", p. 28 (https://books.google.com/books?id=J6BGAAAAcAAJ&pg=PA28).
6. "Cotes, Roger (CTS699R)" (https://venn.lib.cam.ac.uk/cgi-bin/search-2018.pl?sur=&suro=w&fir=&firo=c&cit=&cito=c&c=all&z=all&tex=CTS699R&sye=&eye=&col=all&maxcount=50). *A Cambridge Alumni Database*. University of Cambridge.
7. Edleston, J., ed. (1850) *Correspondence of Sir Isaac Newton and Professor Cotes, …* (London, England: John W. Parker), "Letter XCVIII. Cotes to John Smith." (https://archive.org/stream/correspondenceof00newtrich#page/197/mode/2up) (1708 February 10), pp. 197–200.
8. Kaw, Autar (1 January 2003). "cotes - A Historical Anecdote" (http://mathforcollege.com/nm/anecdotes/cotes.html). *mathforcollege.com*. Retrieved 12 December 2017.
9. O'Connor & Robertson (2005)
10. In *Logometria*, Cotes evaluated e, the base of natural logarithms, to 12 decimal places. See: Roger Cotes (1714) "Logometria," *Philosophical Transactions of the Royal Society of London*, **29** (338) : 5-45; see especially the bottom of page 10. (http://babel.hathitrust.org/cgi/pt?id=ucm.5324351035;view=2up;seq=16) From page 10: *"Porro eadem ratio est inter 2,718281828459 &c et 1, … "* (Furthermore, the same ratio is between 2.718281828459… and 1, …)
11. *Harmonia mensurarum* contains a chapter of comments on Cotes' work by Robert Smith. On page 95, Smith gives the value of 1 radian for the first time. See: Roger Cotes with Robert Smith, ed., *Harmonia mensurarum* … (Cambridge, England: 1722), chapter: Editoris notæ ad Harmoniam mensurarum, top of page 95 (https://books.google.com/books?id=J6BGAAAAcAAJ&pg=RA1-PA95). From page 95: After stating that 180° corresponds to a length of π (3.14159…) along a unit circle (i.e., π radians), Smith writes: *"Unde Modulus Canonis Trigonometrici prodibit 57.2957795130 &c. "* (Whence the conversion factor of trigonometric measure, 57.2957795130… [degrees per radian], will appear.)

12. Roger Cotes with Robert Smith, ed., *Harmonia mensurarum* … (Cambridge, England: 1722), chapter: "Theoremata tum logometrica tum triogonometrica datarum fluxionum fluentes exhibentia, per methodum mensurarum ulterius extensam" (Theorems, some logorithmic, some trigonometric, which yield the fluents of given fluxions by the method of measures further developed), pages 113-114. (https://books.google.com/books?id=J6BGAAAAcAAJ&pg=PA113)
13. Roger Cotes with Robert Smith, ed., *Harmonia mensurarum* … (Cambridge, England: 1722), chapter: "Aestimatio errorum in mixta mathesis per variationes partium trianguli plani et sphaerici" Harmonia mensurarum ... , pages 1-22, see especially page 22. (https://books.google.com/books?id=J6BGAAAAcAAJ&pg=RA1-PA20) From page 22: *"Sit p locus Objecti alicujus ex Observatione prima definitus, … ejus loco tutissime haberi potest."* (Let p be the location of some object defined by observation, q, r, s, the locations of the same object from subsequent observations. Let there also be weights P, Q, R, S reciprocally proportional to the displacements that may arise from the errors in the single observations, and that are given from the given limits of error; and the weights P, Q, R, S are conceived as being placed at p, q, r, s, and their center of gravity Z is found: I say the point Z is the most probable location of the object, and may be most safely had for its true place. [Ronald Gowing, 1983, p. 107])
14. Cotes presented his method in a letter to William Jones, dated 5 May 1716. An excerpt from the letter which discusses the method was published in: [Anon.] (1722), Book review: "An account of a book, intitled, *Harmonia Mensurarum*, … ," *Philosophical Transactions of the Royal Society of London*, **32** : 139-150 ; see pages 146-148. (http://babel.hathitrust.org/cgi/pt?id=ucm.5324350998;view=2up;seq=166)

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External links

- "*Harmonia Mensurarum*" (http://www.mathpages.com/home/kmath192/kmath192.htm). *MathPages*. Retrieved 7 September 2007.- A more complete account of Cotes's involvement with *Principia*, followed by an even more thorough discussion of his mathematical work.
- Roger Cotes (https://mathgenealogy.org/id.php?id=103067) at the Mathematics Genealogy Project

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- Roger Cotes (https://inspirehep.net/author/profile/R.Cotes.1) on INSPIRE-HEP
